My First Dive Into Quantum Computing

Motivation

A Quantum Computer is a machine that utilizes quantum mechanical properties to store and process information. For the past 4 months, I have taken the first steps on my journey to understanding these machines. In this essay, I will articulate the results of my research on a few interesting things I came across the problems quantum computing aims to address, one of the first theoretical applications, the current state-of-the-art technology within the field, a case study that theoretically shows an exponential speed-up in machine learning classification using a quantum computer, and my personal next steps to learning more about the subject.

Why Quantum?

Randomness

A common problem in classical (digital) computing is modeling of randomness, since a true random process by definition **can not** be predicted. That is, no efficient algorithm should be able to predict the output given access to the input and details about the producing system. Classical computers' greatest strength is actually their greatest weakness when it comes to randomness. The strength I am referring to is the high level of precision to which the machine does exactly what is asked of it. This **deterministic** behavior is really great most of the time because it allows us to simulate closed systems and problems without having to correct against errors or noise. Many of the technological developments that have been made in recent decades would not have been possible without this strict and unwavering obediance to instructions that our classical machines possess.

Yet, there exists many applications in which randomness is useful. These applications range from random number generation for lotteries, cryptographic keys, modeling of noise, and modeling of quantum systems.

Simulation

Since Planck's observation of energy absorption and radiation patterns in black-body radiation that could only be explained by quantum mechanics[3], much of the global scientific community has adopted the understanding that the universe we inhabit and the matter it contains can be described most accurately through a quantum mechanical lens. Therefore, an accurate model can not be constructed using only the theories and laws of classical mechanics.

Since quantum computing architectures are build upon and exploit the phenomena explained by quantum mechanics, it is evident that they will be much more effective at modeling a quantum world than their classical counterparts. This modeling, if done correctly, has the potential to vastly increase our understanding of the world around us and can lead to a host of new technological developments and discoveries just as the introduction of classical machines did.

Need for Speed

There are very few problems that can not be solved using a classical computer, and this is amazing! Well at least it would be amazing if we lived forever... This is because even though most problems can be solved on a classical machine, they can not all be solved **efficiently**. If a problem takes 300 years of execution on society's most powerful classical computer to solve, would we really be inclined to say the solution is useful? It is unlikely that the person who initially deployed the algorithm would still be alive to see the answer 300 years later. For this reason, the **computational complexity**, or efficiency, is much more relevant to Computer Scientists than a binary measure of 'computable' or 'not computable'.

The promise that seems to be provided by quantum computers is that of an exponential speed-up for some types of problems.[4][5][6] Although the reason behind the speed-up varies between algorithms, it can most often be attributed to the ability to construct superpositioned states consisting of one or more **qubits** to store information. The result of this is that computers utilizing qubits posess the ability to represent 2^n classical bits with n qubits while classical machines are only capable of storing, (as you might have guessed) n classical bits with n classical bits.

In 1965, Gordon Moore posited a log-linear relationship between device complexity and time [7]. The relationship hypothesized that device complexity (higher circuit density at a reduced cost) would double every two years. This idea, called Moore's Law (misleadingly called 'Law' as it is more of an empirical observation) has recently begun to lose faith among followers. Researchers in the community have recently recognized that we are reaching physical limits to further minutarization of transistors (devices responsible for providing users with programmable classical bits, one transistor per bit). Associated rising costs and reduced return on investment appear to be further slowing the pace of development [8][9]. Most modern classical integrated circuits contain anywhere from 10-35 billion transistors, while Sycamore, Google's state of the art quantum processor, contains 53 Qubits.

The Genesis

Modern Quantum Information Theory can be argued to have been kicked off in 1968 by Stephen Wiesner through his invention of conjugate coding [1]. Conjugate Coding refers to a cryptographic method of information transmission that aims to maintain message integrity and confidentiality. In the study of Cryptography, integrity and confidentiality are two necessary(but not sufficient) conditions that must be satisfied in order to guarantee sound cryptographic security. Integrity refers to the state of the message and its relationship to the initial state of the message. A modified or corrupted message is not said to have integrity. Confidentiality refers to the exclusivity of access to the message, that is, only the intended recipent is able to extract any sort of information from the message in a system that ensures confidentiality.

In order to set the scene for Conjugate coding, we must first examine the **Measurement Problem** and its implications [2]. The measurement problem refers to the effect of observation on a quantum system. While a quantum system can be in a **Superposition** of multiple possible eigenstates (or physical states) at once, the moment an observation on that system takes place, the system collapses onto a single eigenstate and a singular physical state is embodied by the system. When we talk about superposition, the concept we are referring to can be most easily understood by doing away with the intuitive notion of an object being "here" or "there" and instead adopting a probabilistic view of the state. Under this view, we would say that an object can have for example a 50% probability of being "here" and a 50% probabilitiy of being "there". A system in superposition is not said to have a definitive desciption but instead a probabilistic description, nothing more. With this, we can begin to understand one way quantum computers can prove advantageous over their counterparts. In a classical machine, with one bit we have to ultimately define it as either 1 or 0, but with a quantum machine, we can define a qubit to be in a **Superposition** with some probability of being 1 and some probability of being 0. The containment of these two seperate probabilities in a single qubit demonstrates the ability to represent 2^n bits for each qubit, a feat fundamentally impossible with classical computers.

When using Conjugate Coding, a sender who we can call Alice, takes her classical message and encodes it in photons polarized in conjugate basis. This polarization, is responsible for each bit of her message into a superposition. Since photons are the quantum manifestation of electromagnetic radiation, as photons are sent across the wire, they propogate along with two orthogonal wave-like features. These features are the magnetic field component and the electric field component. Upon recieving the message, the recipient, who we will call Bob, passes the photons through a polarizing beam splitter randomly choosing conjugate basis of measurement for each bit. Depending on the basis used, each state has a certain probability of being measured. If Alice and Bob chose the same basis, they are guaranteed to measure the same state with $P = (1.0)$, but if they choose different basis, they can not assume to have the same state for that piece of information. More precisely, they will measure the same state with $P = (.50)$. Since they know this, Alice and Bob can publicly share the basis that they chose and discard from their message block the units of information for which they used different basis. If after pruning their message block in this manner they share the same piece of information, (which they can easily determine by using the message as a symmetric crypographic key and communicating with the key) then Alice and Bob can be confident that their message was not tampered with or measured by any eavesdropper, since an eavesdropper would have to correctly guess each basis that Alice used in order to perfectly recreate the message before sending it to Bob. As the units of information transmitted, denoted by N, grows, the probability that an eavesdropper will correctly guess each basis decreases exponentially. This decrease is represented by the following probability density function, where n represents the length of the message:

$$f(n) = (.50^n)$$

Conjugate coding was just the tip of the iceberg, but it gave us a glimpse into a world where quantum computing can provide new ways of securing our communications, something especially important in the so-called Information Age we happen to find ourselves in.

Where Are We Now?

The current state of the field has matured quite a bit since Weisner's Conjugate Coding invention. This year alone has resulted in a number of incredibly interesting discoveries including:

1. Creation of artificial atoms in silicon quantum dots that contain a higher number of electrons resulting in greater stability for qubits than previously thought possible. [10][11]
2. Presentation of an eight-user city-scale quantum communication network, located in Bristol, using already deployed fibres without active switching or trusted nodes. [12][13]
3. Demonstration of a photonic based Quantum Computer which can perform Gaussian BosonSampling at a rate that is 10^{14} times faster than state-of-the-art quantum simulation using classical supercomputers. [14]

Although progress is speeding along, we are still very much in the NISQ era of quantum computing. A term coined by John Preskill in his 2018 paper[15], NISQ stands for Noisy-Intermediate State Quantum and is meant to illustrate one of the most important aspects of this era, noise. While quantum circuits are beginning to show promising results on certain subsets of computational problems, results become increasingly unreliable as the depth of the circuit (number of operations on a superpositioned system since the last measurement) grows.

Since quantum machines utilize quantum mechanical phenomena such as interference between quantum systems, they are unavoidably affected by other quantum bodies outside the machine. This external interference results in noise and decoherence, that is, the gradual entanglement of the qubit with its environment, and thus lack of preservation of the qubits programmed state. One way to try to address this problem of decoherence is to completely close off the system from outside interference, but one would quickly realize that with a closed quantum system, there would be no way to measure or manipulate the qubits leaving us with a computational machine that we can't perform computations on.

In an era of perfectly error-corrected qubits, capacity of quantum processors may begin to follow a rate of growth similar to that of their classical analogues, unfortunately, we are not in that era yet. For this reason, much of the current research is centered around error and noise correction, with machine learning showing lots of promise for correcting these errors.[16][17][18]

Looking Ahead

In spite of the engineering hurdles that lie ahead, there is still a fair amount of optimism in the field for the future of this technology. With the pressure of the impending possible end to Moore's Law, and the doors that quantum computing can open, it is no wonder that so much research is being done into creating quantum machines that not only have higher numbers of qubits, but also have lower error and decoherence rates.

IBM Quantum has recently published a hardware roadmap that outlines their own goals towards the creation of machines with higher numbers of qubits and lower error rates. Interestingly enough, the relationship between chip complexity and time is roughly the same as Moore's Law. Figure 1 shows the image from their blog post introducing the roadmap.[19]

These are certainly ambitious goals considering the fact that the community has not yet not found a cost-efficient and scalable method to implement complete error-correction, but even if we can't create a fully-scalable quantum machine, there is quite a bit that we can learn from trying.

Quantum-Inspiration

An important part of research that goes into this field is not just the creation of novel quantum algorithms, but something called dequantization. Dequantization of computational problems refers to the demonstration of a contradiction to a claim of quantum supremacy (a quantum computer performing a problem more efficiently or accurately than a classical computer). This is usually done by exploring new classical algorithms for the problem, or creating classical

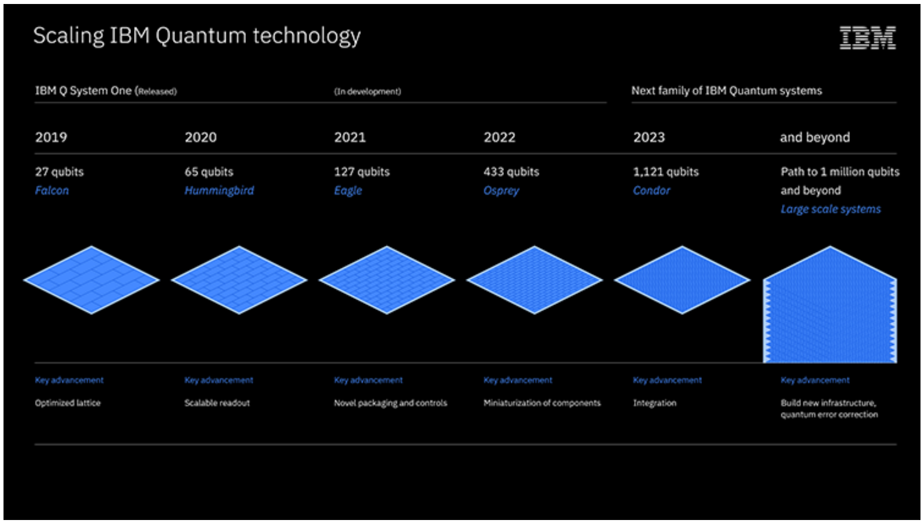


Figure 1: Image from IBM Quantum's storyTK

algorithms that are inspired by their quantum counterpart with higher efficiency than the proposed quantum method. Chia et al. and Tang et al. demonstrated this method in their recent papers[20][21].

In the subset of the multiverse (a discussion for another day) that we never arrive at a period of scalable noise-corrected qubits, there is still benefit to be reaped by the creation of quantum-based algorithms. Although simulating a quantum machine generally requires an allocation of memory that scales exponentially, there has been recent research into ways to maximize efficiency of current classical hardware so that it performs optimally when executing a quantum algorithm.[22] Work like this will greatly increase accessibility to simulating quantum machines and enable the use of quantum algorithms without a quantum machine.

Case Study

I will now explore a specific quantum machine learning algorithm and show how it is possible to acheive an exponential speed-up in unsupervised data classification. Before I jump into how this speed-up is possible, I will briefly cover a few important building blocks used by the algorithm. More in depth resources can be found in these references.[24][25]

Dirac Notation and Qubit Representation

Dirac or Bra-Ket notation is the mathematical notation we utilize to think about and work with qubits.[23] Since qubits and quantum systems in general can exist in a superposition of values before measurement, it is useful to not be limited by a single scalar value to represent the state of the qubit (as would be done in classical computing) but instead a vector representing the **probability amplitudes** for the system existing in each possible state.

$$1 = |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$0 = |0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

In the above equations, we begin on the left-hand side with the classical representation of bits. The middle values are Kets, which can be thought of as column vectors that each represent 1 and 0 respectively. On the right side of the equation is the vector containing the probability amplitudes for each possible state.

Unlike statistical probabilities, probability amplitudes can be negative. Statistical probability for observing a given state can be derived from probability amplitudes using:

$$P = |amplitude|^2$$

These probability amplitudes are often times complex numbers. It is important to note that within literature, a set of probability amplitudes that describe a quantum state are conventionally normalized so that the corresponding statistical probabilities add up to 1. That is:

$$|a_0|^2 + |a_1|^2 + ... + |a_{n-1}|^2 = 1$$

In general, quantum states can be expressed as:

$$|\Psi\rangle = \sum_{i=0}^{N-1} a_i|i\rangle$$

Quantum Gates

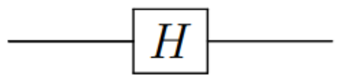
In order to manipulate our qubits, we need to introduce some logical gates similar to those that exist in our classical computing models. These quantum gates come in the form of unitary matrices. An important thing to note about quantum gates that makes them different from classical gates is that they are all reversible. The reason for this is that the conjugate transpose of a unitary matrix is its inverse, and since the inverse of all unitary matrices exist, we can be guaranteed reversibility in gates when using unitary matrices. While there are many other gates, I will only be covering those that are necessary for the algorithm.

Hadamard Gate

One of the most useful gates in circuit-based models of quantum computing is the **Hadamard** gate. It is used for putting a qubit into an equal superposition of $|0\rangle$ and $|1\rangle$.

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

In circuit gate diagramatics it is represented by:



Fredkin Gate

The Fredkin Gate is a controlled swap operation performed on 3 qubits. In other words, The Fredkin gate takes one control qubit and two input qubits, if the control qubit is equal to 1, then the two input qubits are swapped.

$$Fredkin = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

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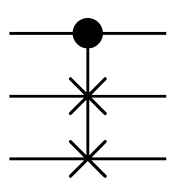


Amplitude Encoding

Amplitude Encoding refers to the encoding of classical information into the probability amplitudes present in quantum systems.[29] Given a real-valued vector, u, Amplitude Encoding converts this classical vector into a quantum state;

$$|\Psi\rangle = \frac{1}{M}\sum_{i=0}^{N-1} u_i|i\rangle$$

In the above equation,M represents the norm of the vector u, which can be given by:

$$M = \sum_{i=0}^{N-1} \sqrt{u_i^2}$$

K-means

K-means is an unsupervised learning algorithm that clusters n observations into K groups for the purpose of data classification without labelling. This algorithm was first proposed by James MacQueen in 1967.[29] The procedure is as follows:

1. Initialize K centroids randomly
2. While centroids continue to change:
   a. Classificiations on all observations are made such that the standard squared error is minimized. In other words, observations are classified into the groups with the closest centroids.
   b. Centroids are re-calculated by taking the arithmetic mean of each cluster.

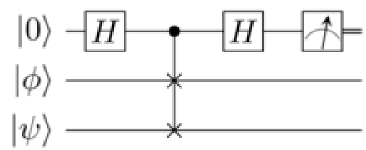
The time complexity of this algorithm is dependent on the values corresponding to the number of features in the input vectors N, the number of input vectors M, and the number of clusters K:

$$O(MNK)$$

Quantum K-means

The Quantum analogue of the classical K-means can be shown to provide an exponential speed-up with respect to the number of features in the input vectors (N). This is made possible through the use of Amplitude Encoding since $logN$ qubits have the capacity to store N-dimensional input vectors. The quantum K-means that I will discuss was proposed by Kopcyzk in his 2018 paper. [30]

One of the most important steps in this algorithm is the **SwapTest**, which was first used by Ameur, Brassard, et al. in their 2006 paper. [31] The SwapTest measures the similarity between two quantum states using Hadamard and Fredkin gates. A diagramtic representation can be found below.



Given two unknown quantum states ϕ and ψ one can measure the similarity or overlap $\langle\phi|\psi\rangle$ by observing the measurement probability of the control qubit in state $|0\rangle$ which is given by:

$$P(|0\rangle) = \frac{1}{2} + \frac{1}{2}|\langle\phi|\psi\rangle$$

Both ϕ and ψ consist of n qubits each and are loaded with classical data using Amplitude Encoding. The resulting overlap, $\langle\phi|\psi\rangle$, is observed by measuring the ancillary qubit, preserving the states of ϕ and ψ.

The general algorithm of the quantum euclidean distance calculation for K-means is as follows:

1. Perform Amplitude Encoding to store our classical vectors a, b into the qubits we want to compare:

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|0,a\rangle) + |1,b\rangle)$$

$$|\phi\rangle = \frac{1}{\sqrt{M}}(|a||0\rangle) + |b||1\rangle)$$

   where $M = |a|^2 + |b|^2$

2. Quantify overlap or similarity between the states using the SwapTest as $\langle\psi|\phi\rangle$
3. Calculate the euclidean distance between the classical vectors using the overlap.

$$Distance = 2M|\langle\psi|\phi\rangle|^2$$

After calculating distances, the closest centroid is found using Grovers Optimization[32] and the observation is classified. Just like in the classical version, there still needs to be recomputation of centroids and the algorithm continues to reiterate the routines of distance calculation, classification, and centroid redefinition until centroid locations cease to change.

Through this example of quantum distance estimation for k-means, it is shown that it is possible to acheive an exponential speed up in efficiency as the number of features for the vectors grows due to Amplitude Encoding. While the classical algorithm acheives a time complexity of $O(NMK)$, the quantum analogue has a complexity of $O(log_2(N)MK)$. A lengthier discussion can be found in Khan's Quantum K-means Algorithm Paper[33].

My Next Steps

These past few months have made me more enthusiastic about pursuing scientific investigation than any other time in my life. At a conference I was at a few weeks ago, the QCWare Q2B Conference, a speaker mentioned that the quantum computing space today feels like what the classical computing space felt like during 1950-1960 period. Having studied the progress that classical machines have made, the thought of getting to be involved in a new technological revolution that is at all comparable to the revolution ushered in by classical computing is beyond exciting. This winter break and over the course of this next semester I plan to embark on two new endeavors to cultivate my knowledge and understanding of quantum computing.

Quantum Computing Collective

The first endeavor is that of founding an undergraduate student organization that provides a shared learning space where students can come learn from, teach, and research with other undergraduates who are interested in quantum computing. This community, which is officially launching Spring 2021, is called the Quantum Computing Collective and is already 51 members strong! In this community we will provide various ways to engage with the literature together, teach eachother how to build quantum circuits using IBM's Qiskit, pursue research in teams with the goal of publication, and host career development events where undergraduates can meet with representatives from Quantum Computing firms and Quantum Computing graduate programs!

Quantum-Classical Financial Volatility Classifier

The second endeavor is a research project that I have begun implementing which will utilize the Quantum K-means algorithm that I presented above to build a quantum-classical financial volatility classification framework. In this research, I will create and compare classical and quantum volatility classification frameworks. The overarching design of the framework is as follows:

1. Data from two categories (market-data and financial news articles) are sourced and pre-processed.
2. Data from the news article category is used to derive a sentiment. This consists of two main parts:
    1. An organizational identifier is used to identify entities and words related to companies in the S&P 500.
    2. Using finBERT, an open source instance of Google's state-of-the-art Deep Learning Natural Language Processing model that is pre-trained on financial data, a sentiment is derived and attached to the entity that was identified and associated with the given time step based on the article timestamp.

3. Data is then transformed into its principal components using Principal Component Analysis for the purpose of reducing the dimensionality of the dataset.
4. Finally, a k-means classifier takes the principal components and classifies the data into k groups.
5. The classification of these environments will allow me to analyze whether or not there exists correlation between group membership and volatility conducivity.**

The first model will be exactly as I described the framework, while the second model will include quantum sub-routines. In step 3, classical PCA will be replaced by Quantum PCA [34], while in step 4, the classical k-means will be replaced with the quantum k-means.

After creating these models, I will perform a 4 year back-test to compare predictive ability and efficiency between the models with the goal of either rejecting or accepting the hypothesis that a quantum volatility model following the above framework can acheive higher accuracy than a classical implementation.

** An important assumption here is that I believe the volatility of today's stock market affects the volatility of tomorrow's stock market. Evidence for this can be seen in non-stationarity of financial time-series data and correlation of realized volatility between consecutive time steps, quantifiable through the use of augmented dickey-fuller tests.

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